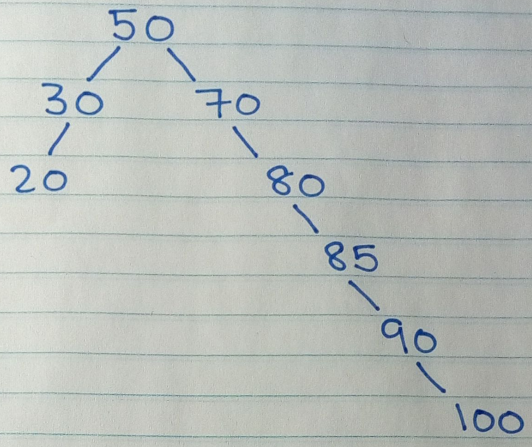
# WBT Rotation Examples

1. Consider the tree below.




70 is right heavy, because $w(70.left) \times 3 < w(70.r)$

$w(70.left) \times 3 = 1 \times 3$
$= 3$

$w(70.right) = 5$

$3 < 5$, so $w(70.left) \times 3 < w(70.right)$.

Now, we check if 70.right is right heavy or left heavy. To do so, we go to the right child of 70, which is 80. 80 is also right heavy as $w(80.left) \times 2 < w(80.right)$.

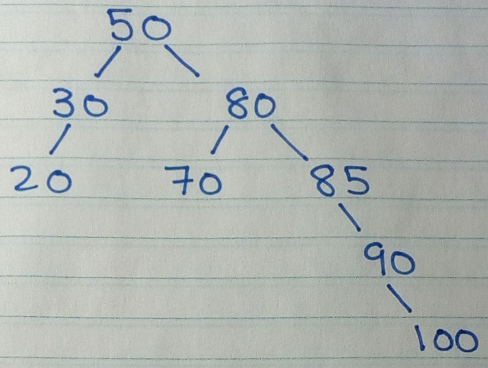
$w(80.left) \times 2 = 1 \times 2$
$= 2$

$w(80.right) = 4$

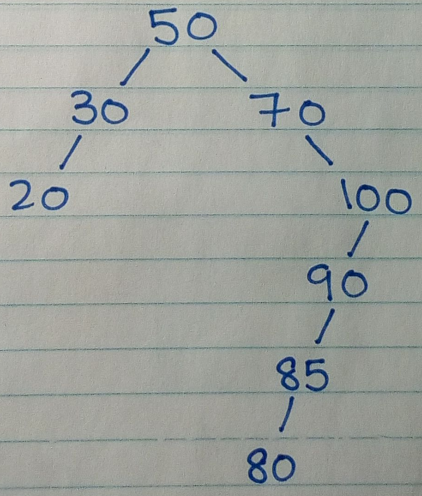
$2 < 4$, so 80 is right heavy.

Since 70 is right heavy and 80 is right heavy, we rotate 70 ccw and bring 80 up.

The final tree looks like:




2. Consider the tree below.



Here, 70 is right heavy, because $w(70.\text{left}) \times 3 < w(70.\text{right})$.

$w(70.\text{left}) \times 3 = 1 \times 3$
$= 3$

$w(70.\text{right}) = 5$

$3 < 5$, so 70 is right heavy.

Now, we go to 70's right child to see if it's left heavy or right heavy. 70's right child is 100 and it is left heavy.

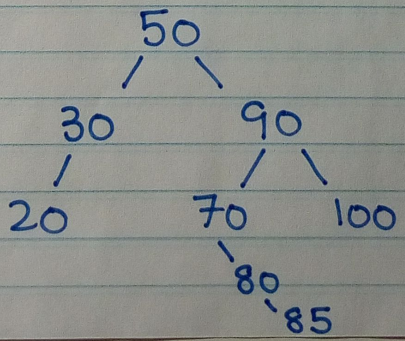
$w(100.\text{left}) \times 2 = 8$

$w(100.\text{right}) = 1$

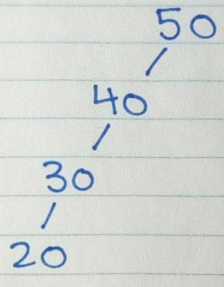
$8 > 1$, so 100 is left heavy.

Since 70 is right heavy and 100 is left heavy, we need a double rotation. First, we rotate 100 cw, bringing 90 up. Then, we rotate 70 ccw.

The final tree looks like:

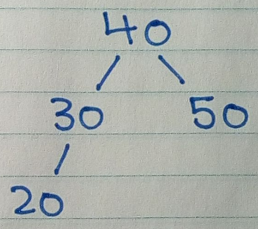


3. Consider the tree below.

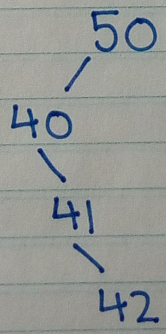



50 is left heavy. Now, we go to the left child of 50 to see if it's left or right heavy. 40 is left heavy, so we rotate 50 cw and bring 40 up.

The final tree looks like this:



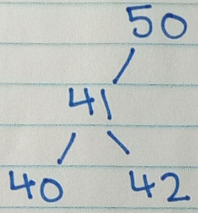

4. Consider the graph below.



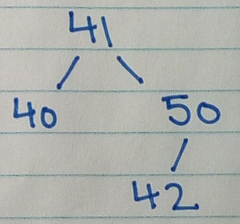
50 is left heavy, so we go to its left child to see if the left child of 50 is left heavy or right heavy. 40 is right heavy. Therefore, we need to do a double rotation. First, we rotate 40 ccw and bring 41 up. Then, we rotate 50 cw and make 41 the new root.

The process is shown below.

1.




2.




The final tree

This is the general algorithm for balancing WBTs:

Let v be the current node.

```
if (w(v.left) x3 < w(v.right)):
    let x = v.right

    if (w(x.left)  < w(x.right)  x2):
        rotate ccw about v

    else:
        rotate cw about x then ccw about v


else if (w(v.right) x3 < w(v.left)):
    let x = v.left

    if (w(x.right)  < w(x.left)  x2):
        rotate cw about v

    else:
        rotate ccw about x then cw about v

else:
  Do nothing
```

In general, there are 4 cases to look for.

1. Left heavy, left heavy

   For this case, v is left heavy and when we let x = v.left, x is also left heavy. Here, we rotate cw about v. Look at example 3.

2. Left heavy, right heavy

   For this case, v is left heavy and when we let x = v.left, x is right heavy. In this case, we rotate ccw about x and then cw about v. Look at example 4.

3. Right heavy, right heavy

   For this case, v is right heavy and when we let x = v.right, x is also right heavy. Here, we rotate ccw about v. Look at example 1.

4. Right heavy, Left heavy

   For this case, v is right heavy and when we let x = v.right, x is left heavy. Here, we rotate cw about x then ccw about v. Look at example 2.